Self-organized, globally optimal, and real-world urban transportation networks – Similarities and discrepancies

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Abstract

Current transportation network design approaches, whether from engineering, economics, or the social sciences, predominantly follow traditional top-down strategies. Yet cities are increasingly recognized as complex systems, where transportation networks evolve through dynamic feedback loops: the network shapes origin-destination (OD) patterns (top-down), while OD flows adapt and feed back into network evolution (bottom-up). However, research on such bottom-up planning remains limited, and existing studies often assume user equilibrium without considering individual preferences and participation. In this work, we present a data-driven framework to analyze and compare three types of transportation networks: self-organized, globally optimal, and real-world. We compare these networks geometrically and use optimal transport theory to analyze their similarities and differences. Using the Singapore transport network as a case study, we find that the existing network more closely resembles a self-organized network than a globally optimal one. Our framework can be readily adapted to various regions, offering a tool for assessing design–use misalignments and supporting more adaptive infrastructure planning.

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ABSTRACT

Current transportation network design approaches, whether from engineering, economics, or the social sciences, predominantly follow traditional top-down strategies. Yet cities are increasingly recognized as complex systems, where transportation networks evolve through dynamic feedback loops: the network shapes origin-destination (OD) patterns (top-down), while OD flows adapt and feed back into network evolution (bottom-up). However, research on such bottom-up planning remains limited, and existing studies often assume user equilibrium without considering individual preferences and participation. In this work, we present a data-driven framework to analyze and compare three types of transportation networks: self-organized, globally optimal, and real-world. We compare these networks geometrically and use optimal transport theory to analyze their similarities and differences. Using the Singapore transport network as a case study, we find that the existing network more closely resembles a self-organized network than a globally optimal one. Our framework can be readily adapted to various regions, offering a tool for assessing design–use misalignments and supporting more adaptive infrastructure planning.

Keywords: Bottom-up planning, Self-organized network, Globally optimal network, Optimal transport theory;

INTRODUCTION

The urban transportation network is the backbone of the city, which connects the various components of the city and has a crucial impact on the economic development of the city and the daily life of its residents. It has evolved over time and with advancements in technology and transitions in societal needs and preferences, and shows variegated patterns in different regions, reflecting diverse geographical, economic, and cultural characteristics (*1, 2*). This evolution highlights the complexity of transportation networks in satisfying the diverse needs of communities.

Current transportation network design is normally contingent on the multi-perspective optimal performance of transportation networks. Scholars in the field of transportation pay numerous efforts to seeking optimal transportation network configuration with respect to minimal travel distance or time, maximal accessibility with limited cost (*3, 4*), and various constraints. Economists and operation research scholars prefer the best return on investment and concentrate on the implementation of road pricing to efficiently alleviate congestion externalities (*5*). Multi-objective optimal problem is also ubiquitously proposed to simultaneously minimize costs, time and emission (*6–9*). There are also numerous works constructed bi-level models, while the most prevalent form of the lower level problem considers the route choice of the users, which is called the trip assignment problem, and the upper level is normally a design problem to achieve the optimal objective function (*10*).

In addition, sociologists show enormous interest in human behaviors in transportation systems (*11*). Previous works disclosed the conformity tendency and route choice of human mobility behaviors under different scenarios (*12, 13*). (*14*) and (*15*) explored the changes in human behaviors responding to the rewards to avoid rush-hour congestion by long-term field study. Nevertheless, the subjective human behavior in transportation networks is often difficult to predict accurately and can only be roughly estimated by drawing on findings from sociology, anthropology, cognitive psychology, and brain science (*11*), which rely on behavioral experiments, large quantities of expensive questionnaires and datasets (*16*). However, the aforementioned human behavior analysis is also conducted based on the current transportation structures and proposed top-bottom strategies to regulate human behaviors. While, the study of (*17*) pointed out that in some situations, the existing formal structures do not meet individual or group needs, this will lead to the appearance of a desire path. Urban theorists and community activists view desire lines as a form of resistance and reclamation of space for a public poorly served by urban institutions (*18*).

Most current approaches to transportation network design, not only from the perspective of engineers and economists, but also from the perspective of sociologists, aim at the traditional top-down strategies (*1*), where decisions, plans, or policies are initiated at the highest level of an organization or system, and then passed down through the hierarchy to be implemented at lower levels (*19*). Recent research pointed out that cities are prototypical complex systems (*20, 21*), emphasizing the bottom-up evolution from numerous interacting components whose behaviors and interactions to the system's overall complexity and emergent properties. The transportation system is a critical component and complex system for each city, evolving through a feedback loop as the transportation network will influence the OD pattern (top-down) and then the OD pattern will also provide feedback to the transportation system (bottom-up). Consequentially, the multi-dimensional and interdisciplinary study of human behavior and transportation engineering, as well as operations research, is essential to construct an efficient and comprehensive transportation network. Urban transportation network design as a dynamic feedback loop, bottom-up strategies, and methods should be more widely studied to enrich current approaches to city planning and benefit all city dwellers. However, few studies are concentrating on bottom-up transportation network planning. Besides, this kind of post-optimization typically presupposes that individuals reach a state of user equilibrium following the establishment or alteration of the current state while overlooking people's preferences and participation in the existing situation (*22*).

In this work, we introduce a data-driven modeling framework to study the similarities and discrepancies among *i*) self-organized, *ii*) globally optimal, and *iii*) real-world transportation networks. The minimal model for the self-organized network is motivated by the bottom-up formation of human trail networks (*23*). It balances the human preference for taking the shortest path and the propensity to share popular route segments with others (e.g., for increased travel speed and comfort, access to amenities) (*24*), thus encapsulating the inherent tendencies of navigation and social behavior. The globally optimal transportation network follows the recent work by Bontorin et al. (*25*). It is obtained by constructing a two-layer model that minimizes the total travel time for a fixed infrastructure cost, using link width (reflecting the travel speed on a given path segment) as the decision variable. In the lower layer, individuals will choose the minimal travel time path between their origin and destination, while the role of the upper layer is to minimize the objective function, i.e., the total travel time summed over all trips. Both network models are derived purely based on origin-destination (OD) trips of urban populations and do not consider existing transportation systems. Subsequently, the most straightforward network comparison follows from a geometric perspective. In addition, we apply principles from optimal transport theory to analyze and comprehend the geometry of functional spaces (*19*).

The contribution of this work is threefold. First, we introduce a data-driven, bottom-up framework to reflect the consistency and discrepancy between realistic, optimized, and human-desired transportation networks. Utilizing Singapore as a case study, this approach offers robust applicability to other regions. The output of this framework could be used to scrutinize user satisfaction with existing transportation networks from a bottom-up perspective, highlight the limitations of the current network, and furnish novel insights for transportation network design. Moreover, this framework aids in assessing the state of urban transportation networks and providing governments and planners with strategies to achieve smart and green cities. Secondly, previous studies tend to focus on specific factors such as road capacity, emission, congestion time, etc., and the pivotal importance of network topology has been neglected; we extract and compare the topologies of the transportation networks for each case, which can help to do further complex network analysis such as resilience and other characteristics. Finally, this study is not limited to specific transportation network types, such as a road or public transit networks. Therefore, it can be used not only for traditional transportation infrastructure allocation problems (e.g., placement of bus stops and subway stations), but also for the infrastructure construction of new technologies, such as air taxi routes, connected vehicles, and automated vehicle lanes, thus providing an integrated tool for future transportation planning.

The structure of this paper is as follows. Firstly, we introduce the concept of a self-organized transportation network based on shortest path and trajectory bundling techniques. Subsequently, we construct a prior-optimized transportation network using optimal transport theory to facilitate comparisons of the established self-organized network, and globally optimal network with existing real-world transport networks. Using Singapore as a case study, we then apply our framework to a comprehensive dataset derived from mobile phone GPS data. Finally, we present the conclusions and discuss the limitations of our work.

SELF-ORGANIZED TRANSPORTATION NETWORK

Transportation networks encompass multiple layers, including public transportation, highways, and sidewalks. Previous research has focused on existing physical infrastructure, but the practical human usage patterns do not always align with these predetermined structures. For example, despite the existence of designated paths, people often create shortcuts through grassy areas, known as “desire paths" (*17, 26*). The self-organized paths based on human behavioral science and simulation have traditionally been studied in the context of pedestrian or cycling movement (*27*) or informal paratransit services with ad-hoc routes (*28*). We extend this desired path concept to the entire transportation network, in which we do not consider practical constraints such as the existence of roads, but instead focus on modeling travel routes based on human behavioral principles.

Self-organized network construction

Transportation network demonstrates self-organization because it is a complex system where simple interactions at the individual level can lead to complex but organized patterns at the system level (*20, 29*). Human behaviors in transportation networks have been extensively investigated in macro-scope such as the gravity law (*30*), visitation law (*24*), and radiation model (*31*). Individual behaviors are normally rational, drawing on findings from sociology, anthropology, cognitive psychology, and brain science (*11*). At the core of human behavior in transportation is the pursuit of “reward-seeking". That is, individuals naturally gravitate towards options that offer pleasure or benefits while minimizing harm or discomfort. For example, the shortest and fastest route is typically preferred by travelers (*27*). Another characteristic of human behavior in transportation is social conformity. This phenomenon, where individuals adjust their behaviors, attitudes, or beliefs to match those of a group, often due to real or perceived group pressure, plays a crucial role in decision-making and navigation processes. Studies such as (*32, 33*) demonstrate that people are more inclined to visit places that attract larger crowds. In transportation studies, car-following and platooning behaviors are commonly seen and used to simulate vehicle behaviors in motorized traffic, which can also be regarded as the conformity of motorized traffic (*34*). These behavioral tendencies can be integrated into human behavior modeling in transportation as follows:

Shortest path

Assuming that the total origin-destination (OD) demand is represented as $OD\{(\mathbf{o_1},\mathbf{d_1}),(\mathbf{o_2},\mathbf{d_2})...,(\mathbf{o_N},\mathbf{d_N})\}$, encompassing N OD pairs, where $\mathbf{o_i}$ and $\mathbf{d_i}$ are the coordinates of origin and destination denoted by longitude and latitude, (x_i, y_i). The route chosen for each OD pair is denoted by $Route\{\mathbf{r_1},\mathbf{r_2},...\}$, where $\mathbf{r_i}$ is a sequence of locations and denotes the route from $\mathbf{o_i}$ to $\mathbf{d_i}$. The rewards, such as time cost, energy consumption, and carbon emissions, are positively proportional to the total distance of $\mathbf{r_i}$ when disregarding the impact of the road network and other regulatory factors. Apparently, the shortest path, $\mathbf{r_i} = \mathbf{o_i} \rightarrow \mathbf{d_i}$ with distance $||\mathbf{o_i} - \mathbf{d_i}||_2$, embodies the maximum rewards, reflecting the reward-seeking characteristic of human behaviors.

Trajectory bundling

Considering the human behavioral conformity and self-organizing behaviors, which indicates that individuals prefer to follow others during the trip, we can use the partition-group trajectory clustering framework developed in (*35*) to simulate this process. At initial, the route choice depends on the shortest path principle and is expressed as $\mathbf{r_i} = \mathbf{o_i} \rightarrow \mathbf{d_i}$. The whole route could be partitioned by sampling distance S into $\lceil \frac{||o_i-d_i||_2}{S} \rceil$ pieces, and then the route is discretized into

$\mathbf{r_i} = \mathbf{o_i} \rightarrow \mathbf{m_i^1} \rightarrow \mathbf{m_i^2} ... \rightarrow \mathbf{m_i^{\lceil \frac{||o_i - d_i||_2}{S} \rceil - 1}} \rightarrow \mathbf{d_i}$, where $\mathbf{m_i^j}$ is the intermediate points for the whole route. At that time, the number of points involved in the total N OD pairs is P, where

$$P = \sum_{i=1}^{N} \lceil \frac{||o_i - d_i||_2}{S} \rceil - 1 \tag{1}$$

The 2-dimensional weighted kernel density estimation (KDE), which is an extension of the standard kernel density estimation, allows individual data points to contribute differently to the estimated probability density function (PDF) based on their frequency. The formula for a two-dimensional weighted KDE is given by:

$$\begin{aligned} \hat{f}(x,y) &= \frac{1}{P} \sum_{i=1}^{N} \sum_{j=1}^{\lceil \frac{||o_i - d_i||_2}{S} \rceil - 1} w_{OD}^{i} K_h(x - x_i^j, y - y_i^j) \\ &= \frac{1}{Ph^2} \sum_{i=1}^{N} \sum_{j=1}^{\lceil \frac{||o_i - d_i||_2}{S} \rceil - 1} w_{OD}^{i} K\left(\frac{x - x_i^j}{h}, \frac{y - y_i^j}{h}\right) \end{aligned} \tag{2}$$

where $\hat{f}(x,y)$ represents the probability density function of location(x,y). K is a two-dimensional kernel function, which is often the product of one-dimensional kernel functions. K_h is a two-dimensional kernel function with bandwidth h, commonly defined as $K_h(x,y) = \frac{1}{h^2} K\left(\frac{x}{h}, \frac{y}{h}\right)$. h is the bandwidth parameter, controlling the width of the kernel function . In this formulation, each data point's contribution to the density estimate is adjusted by its frequency of occurrence w_i in the whole dataset. $N_{wOD} = \sum_{i=1}^{P} w_{OD}^{i}$ is the sum of all weights. This weighted approach allows for a more nuanced density estimation, especially useful in datasets where certain observations are more significant or have different frequencies. As mentioned, individuals tend to move following others, hence the intermediate points move towards the densest location, this kind of human behavior simulation is called point advection, which has been extensively used in previous literature to visualize the urban arterial road (*36*).

For each intermediate point $\mathbf{m}(x,y)$ with a density estimate $\hat{f}(x,y)$, the new position (x', y') of the point after moving a step size η along the gradient of the density estimate can be expressed as:

$$x' = x + \eta \cdot \frac{\partial \hat{f}}{\partial x}(x,y)$$

$$y' = y + \eta \cdot \frac{\partial \hat{f}}{\partial y}(x,y)$$

where $\frac{\partial \hat{f}}{\partial x}(x,y)$ and $\frac{\partial \hat{f}}{\partial y}(x,y)$ are the gradients of $\hat{f}$ at (x,y) in the x and y directions, respectively. Consequently, the reconstructed route for each OD demand is updated to $\mathbf{r_i'} = \mathbf{o_i} \rightarrow \mathbf{m_i^{1'}} \rightarrow \mathbf{m_i^{2'}} ... \rightarrow \mathbf{m_i^{\lceil \frac{||o_i - d_i||_2}{S} \rceil - 1'}} \rightarrow \mathbf{d_i}$. At that time, the KDE should be performed again to the reconstruction route, and the new probability density function is $\hat{f}'(x,y)$. The total variation distance (TVD) between $\hat{f}(x,y)$ and $\hat{f}'(x,y)$ is calculated as,

$$TVD(\hat{F}, \hat{F}') = \frac{1}{2} \int \int |\hat{f}(x,y) - \hat{f}'(x,y)| \, dx \, dy$$

If TVD is larger than a certain threshold T, it means the current routes do not achieve the converged state and the point should be advected again. The intermediate points advection should stop when TVD is smaller than the specific threshold, thereby avoiding overconverge. Algorithm

1 shows the detailed procedure and iteration process for the trajectory clustering.

Algorithm 1 Desire-path Extraction

Initialize Origin-Destination Demand O_i and D_i
for all O_i, D_i **do**
  Connect O_i to D_i directly to form Path P_i ▷ Construct shortest path
end for
while convergence not achieved **do**
  for all R_i **do**
    Sample P_i into points $Point_i$ at fixed intervals l
  end for
  for all $Point_i$ **do**
    Perform weighted Kernel Density Estimation (KDE)
  end for
  for all $Point_i$ **do**
    Adjust non-terminal points slightly towards the weighted KDE gradient direction
    Apply Laplacian smoothing to $Point_i$ to form $ReconstructedPath_i$
  end for
  Calculate TVD between $ReconstructedPath_i$ and $Path_i$
  if TVD $<$ Threshold **then**
    Break ▷ Convergence achieved
  end if
  Set $R_i = ReconstructedPath_i$
end while

Network extraction

It is supposed that after trajectory bundling, the individual routes are estimated as $\mathbf{r_i'} = \mathbf{o_i} \to \mathbf{m_i^{1'}} \to \mathbf{m_i^{2'}} \ldots \to \mathbf{m_i^{\lceil \frac{||o_i - d_i||_2}{S} \rceil - 1'}} \to \mathbf{d_i}$, the objective is to extract the main self-organized network from these aggregated routes. We hypothesize that the final network can be modeled as a graph $G_h(V,E)$, where V denotes the set of nodes and E represents the set of edges connecting these nodes. To achieve a clearer human-desired network without the confusion of overlapping paths, kernel density estimation results are incorporated to select the most popular sites, which also serve as nodes within the graph. So the vertices set $V = \{\mathbf{c_1}, \mathbf{c_2} \ldots\}$ of the graph G_h is chosen as

$$V = \{\mathbf{c} = (x,y) \,|\, f(x,y) > T_d\}$$

where T_d is the density threshold to filter the sparsely populated location. All nodes are used as the clustering centers to classify all intermediate points $m_i'^j \in r_i'(i = 1,2 \ldots N)$, $m_i'^j$ is assigned to c_k if the Euclidean distance between $m_i'^j$ and c_k is equal to $min_{c_k \in V} d(m_i'^j, c_k)$. Then the intermediate points in all routes are replaced by the belonged clustering center position and compose the new route r''. At that time, all the routes r'' only contain the clustering center locations by specific sequence. The edge e_{kl} in E exists if $c_k \to c_l$ is a part of r'', and the edge weight is defined as the number of occurrences of $c_k \rightarrow c_l$. Finally, the built-up graph $G_h(V,E)$ explicitly shows the self-organized network.

GLOBALLY OPTIMAL TRANSPORTATION NETWORK

In this section, the prior-optimal transportation network is constructed. It is worth mentioning that previous studies concentrated on the improvement of existing transportation networks to achieve certain optimal objectives, which is called post-optimization. However, this section shifts focus to optimizing the transportation network, setting aside the existing transportation infrastructure. The reason is that this framework is aimed to provide bottom-up insights, the network should be driven by the OD demand thereby obtaining the bottom-up feedback to refine the dynamic process of urban transport planning.

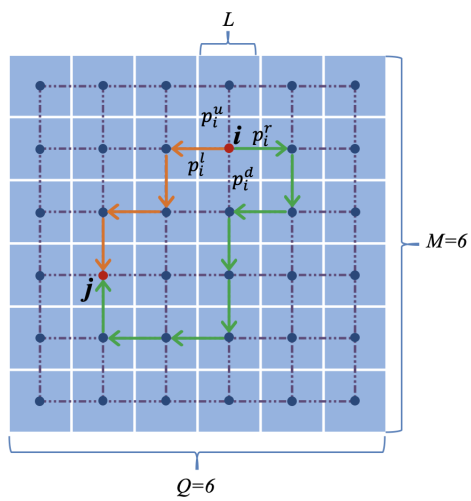


FIGURE 1: Example of a spatial gridded network

In order to simplify this problem and reduce the computational complexity, gridded geolocation is necessary. As shown in Fig. 1, in the initialization step, the whole study area is gridded into $M \times Q$ grids with size $L \times L$. The grid in row m and column q, where $1 < m \leq M, 1 < q \leq Q$, is labeled as $g_i(x_i, y_i)$, $i = mQ - Q + q$, and x_i, y_i represent the longitude and latitude of the center of grid respectively. For each grid, there are four edges of length L linking the four neighbor grids, hence the edge set $E\{e_1, e_2, ...\}$ contains $2MQ - M - Q$ edges and the corresponding weight for each edge is $W\{w_1, w_2, ...\}$, which indicates a velocity of the edge in the transportation network with constraints as $0 \leq w_k \leq T_w$. The travel time t_i for edge e_i is calculated as $\frac{L}{w_k}$. The cost for edge e_i is conceptualized as $a{w_k}^b + c$, where a, b, and c are constants that account for material costs, labor, land acquisition, and other factors. Given $OD\{(\mathbf{o_1}, \mathbf{d_1}), (\mathbf{o_2}, \mathbf{d_2}) ..., (\mathbf{o_N}, \mathbf{d_N})\}$, all $\mathbf{o_i}$ and $\mathbf{d_i}$ are normalized to grid labels as $OD\{(g_i, g_j)\}$. P_{ij} contains all possible paths from g_i to g_j. For example, as shown in Fig. 1, there are two possible paths from g_i to g_j. The number of origin-destination demand from g_i to g_j is represented as OD_{ij}. Each OD demand will be assigned the paths based on minimal travel time criteria, it could be considered as a shortest path selection problem of P_{ij}, which is formulated as,

$$T_{ij} = \min_{\forall p_{ij} \in P_{ij}} \sum_{\forall w_k \in p_{ij}} \frac{L}{w_k} \quad (3)$$

The objective function is defined as the minimal total travel time,

$$\min \sum_{i=1}^{N} \sum_{\forall j \neq i}^{N} OD_{ij} T_{ij}$$

The total cost is expressed as $Cost = (2MQ - M - Q)c + a\sum_{k=1}^{2MQ-M-Q} {w_k}^b$. The total cost is subjected to a specific threshold T or also could be set as a objective function. Simulated annealing algorithm with the shortest path could solve this non-linear optimization problem

COMPARISON OF TRANSPORTATION NETWORKS BASED ON OPTIMAL TRANSPORT THEORY

The most intuitive comparison arises from a geometric perspective. The study area could be partitioned into regions. Each region possesses a distinct linear representation of its topology, allowing the calculation of the Hausdorff distance to characterize the geometric similarity between different networks. The Hausdorff distance is a measure of the similarity between two sets of points in a metric space (*37*). It quantifies the maximum distance from any point in one set to the nearest point in the other set, and vice versa. It is commonly used in computer vision, image processing, pattern recognition, and computational geometry for comparing shapes, objects, or point clouds (*38*).

Let A and B be two sets of sample points belonging to two topologies in a metric space X. The Hausdorff distance $d_H(A,B)$ between sets A and B is defined as:

$$d_H(A,B) = \max \left\{ \sup_{a \in A} \inf_{b \in B} d(a,b), \sup_{b \in B} \inf_{a \in A} d(b,a) \right\} \quad (4)$$

where $d(a,b)$ represents the distance between points a and b in the metric space X, and sup and inf denote the supremum (least upper bound) and infimum (greatest lower bound), respectively.

While it is possible to visualize the topology of the resulting transportation network for intuitive geometry comparisons with existing transportation networks, as well as simple comparisons with information such as the number of nodes and loops, a global measure is still missing. Optimal transport is a fundamental framework for analyzing and understanding the geometry of functional space, as well as a means of interpolating between different functions (*39*). It can help understand the minimum 'cost' of moving from one network topology to another, thus reflecting the similarities and differences between different network topologies. Optimal transport is a field with an active research community in both mathematical theory and engineering applications, such as geometric processing and machine learning (*40*).

Assume the existing real-world transportation network is denoted as $G_r(V_r, E_r)$, and the network to be compared is denoted as $G_h(V_h, E_h)$, where V_r and V_h represent the sets of nodes, and E_r and E_h represent the sets of edges for the respective networks. It is specified that the set V_r contains $|V_r|$ elements, with each element representing a node characterized by its longitude and latitude, denoted as $\mathbf{n_r}(x_r, y_r)$. The set E_r comprises $|E_r|$ edges, with each edge represented as $\mathbf{e_r}(n_r^i, n_r^j, w_r)$, signifying an edge from node n_r^i to node n_r^j with a weight of w_r in the network G_r. The edge information also could be converted to adjacent matrix $Gamma_r$ where $\Gamma(i,j)$ is equal to w_r in $\mathbf{e_r}(n_r^i, n_r^j, w_r)$.

The cost function $\Pi(p,q)$ is defined as the distance from the pth node in G_r to the qth node in G_h , which could be formulated as

$$\Pi(p,q) = ||\mathbf{n_r^p} - \mathbf{n_h^q}||_2 \quad (5)$$

The next step is to convert the structure of a network topology into a probability distribution according to the extracted network features. A typical approach is to use a node's centrality metrics (e.g., degree centrality, proximity centrality, median centrality, etc.) to indicate the importance of a node. The higher the centrality of a node, the greater the importance of that node in the network. Hence, the degree of centrality is chosen as the basis for a probability distribution in which the probability of each node is proportional to its centrality metric. Link weights can also be used as a basis for probability distributions, taking into account that each link in a transport network may serve different sizes of traffic flow. The centrality metric cen of G_h and G_r could be calculated as,

$$cen(n_r^i) = \sum_{j=1}^{|V_r|} \Gamma_r(i,j), \text{ and } cen(n_h^i) = \sum_{j=1}^{|V_h|} \Gamma_h(i,j) \tag{6}$$

The normalized probability distributions of centrality metric for G_r andG_h are ,

$$p_r^i = \frac{cen(n_r^i)}{\sum_{j=1}^{|V_r^j|} cen(n_r^i)}, \text{and } p_h^i = \frac{cen(n_h^i)}{\sum_{j=1}^{|V_r^j|} cen(n_h^i)}$$

Our goal is to minimize the total transport cost of transferring the network to another one, and the minimal transport cost is a global measurement to evaluate the difference between two networks with respect to functional distribution:

$$\min_{\gamma} \sum_{i \in p_r, j \in p_h} \Pi_{ij} \gamma_{ij} \tag{7}$$

where γ_{ij} represents the mass transported from the i-th element of p_r to the j-th element of p_h, and Π_{ij} is the corresponding cost.

The transport plan needs to satisfy the marginal distribution constraints to ensure mass conservation. For each element in p_r or p_h, the total mass transported out equals its mass. Additionally, every element in the transport plan must be non-negative. The constraints are summarized as follows:

$$\sum_j \gamma_{ij} = p_r^i, \quad \forall i \sum_i \gamma_{ij} \quad = p_h^j, \quad \forall j \gamma_{ij} \geq 0, \quad \forall i,j \tag{8}$$

This formulation could be easily solved by linear programming.

CASE STUDY

We extend in this section the application of the framework by integrating data from a real urban structure. In this paper, we utilize two distinct datasets to analyze the similarities and differences of the transportation networks of Singapore: an individual-level GPS-based mobile phone dataset from citydata.ai (*41*) and an aggregated OD dataset for trips by Mass Rapid Transit (MRT) from the Singapore Land Transport Agency (LTA) Datamall (*42*).

Data Description

The initial dataset from CityData.ai, based on GPS from mobile phone devices, encompasses data from 894,327 users collected from September 1 to September 30, 2019, in Singapore. Due to the varying sampling rates of this dataset, selecting active users is necessary. Criteria for selecting active users included a minimum appearance of 5 days in the dataset and over 120 sampling records throughout the month (*43*). This selection process resulted in 244,515 active users and 218,248,627 records. To infer OD demand, the area of Singapore was divided into grid cells with a size of $500m \times 500m$. For each individual, a minimum stay time of 30 minutes, during which a user needs to stay inside a given grid cell to be counted as a valid stop. Subsequently, the sequence of stops for each user was used to compose the total 3,932,249 OD demand with 1,005,396 unique ODs.

Nevertheless, considering the fact that urban transportation networks are characterized by multiple layers, including pedestrian pathways, cycling paths, highways, etc. At the moment of

OD extraction, the grid cells are predefined, making it impractical to account for the walking trip network. To streamline the analysis and reduce computational complexity, OD pairs shorter than 1 km and those appearing fewer than three times in the entire month's data are filtered out, thereby emphasizing longer-distance travels. After filtering, there are still 2,762,943 OD demands with 225,392 unique OD. Fig. 2 illustrates the number of trips for each location.

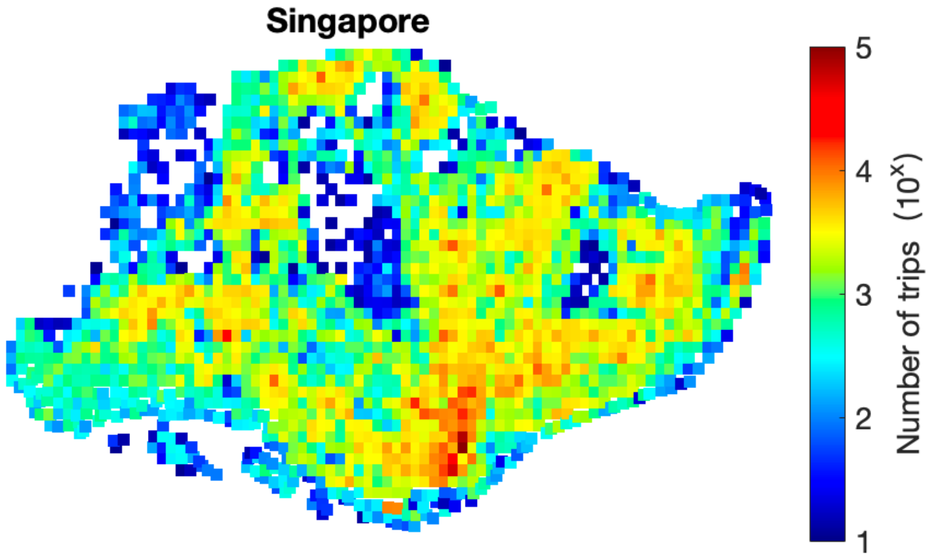


FIGURE 2: Trips heatmap extracted from GPS data

The OD data from LTA Datamall offers the number of Mass Rapid Transit (MRT) trips from origin to destination stations, segmented by weekdays and weekends for each hour. For our study, we selected December 2023 as the period of interest, during which there were 76,610,749 MRT trips with 20,223 unique OD train stations recorded. Adhering to the principle of the shortest path with minimal interchanges, the stations traversed by each origin-destination (OD) pair can be determined based on the MRT network. Fig. 3 visualizes the main MRT and LRT networks with passenger volume in Singapore.

Self-organized Transportation Network in Singapore

We adopted the trajectory bundling technical mentioned above to simulate the human's self-organized pattern. Fig. 4 shows an example of the initial OD pairs and after trajectory bundling. Please see the Appendix for the detailed transition for the one-month OD data in the trajectory bundling process. Algorithm 1 is adopted to construct the self-organized network based on trajectory after bundling. Then we set the top 10% locations with the highest density as the clustering nodes. The final self-organized network is shown in Fig. 5. The trade-off between the converge rate and the maximal allowable permissible percentage of bypasses (shortest path and conformity trade-off) should be considered.

Optimal transportation network in Singapore

In this case study, to reduce the computational complexity, the whole Singapore region into grids with $L = 1.5km$. The grid size could be smaller in the future. The minimal total travel time network is as shown in Fig. 6.

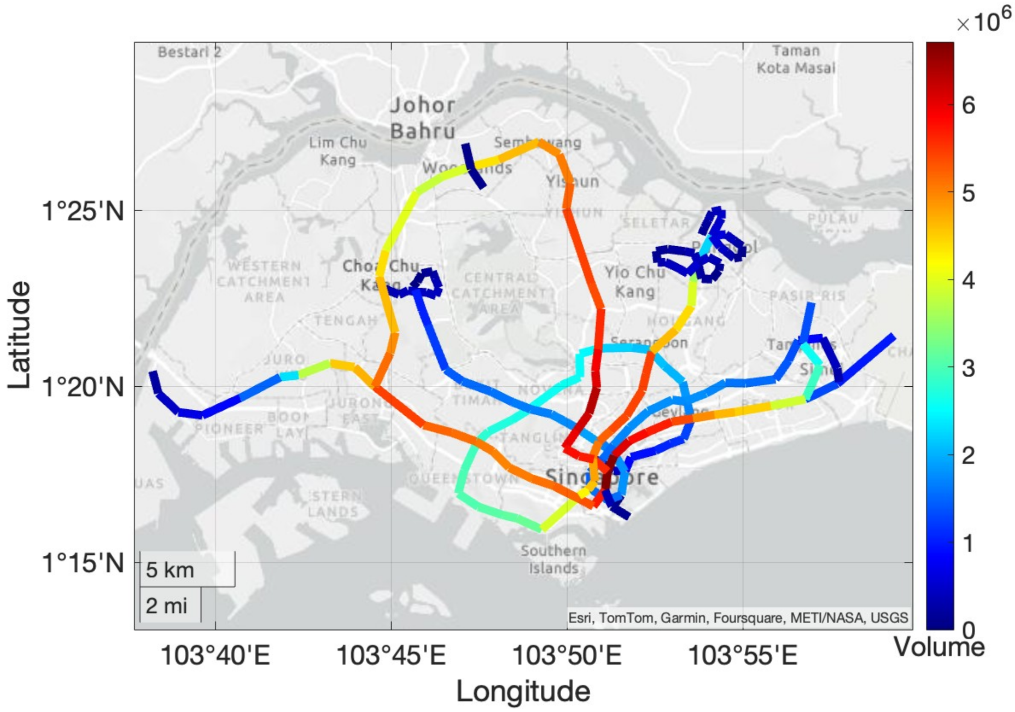


FIGURE 3: Singapore MRT and LRT network with passenger volume

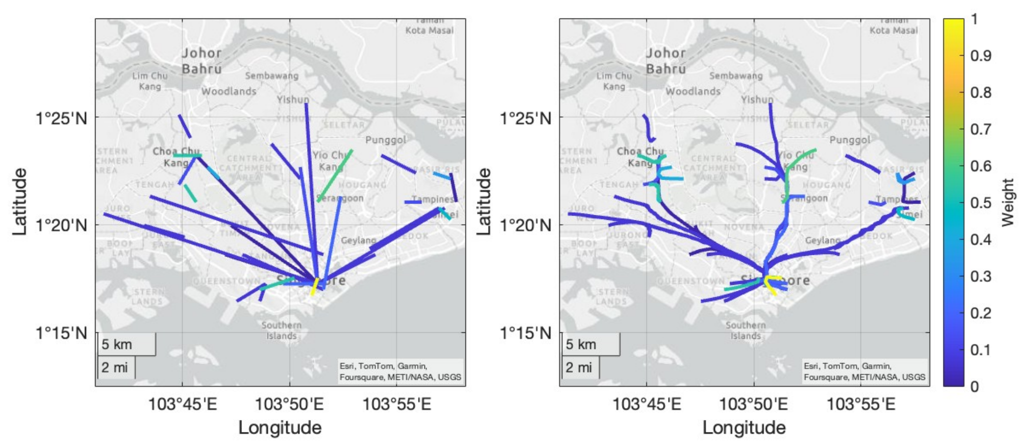


FIGURE 4: The self-organized network modelling; Left: the initial OD; Right: trajectory after bundling

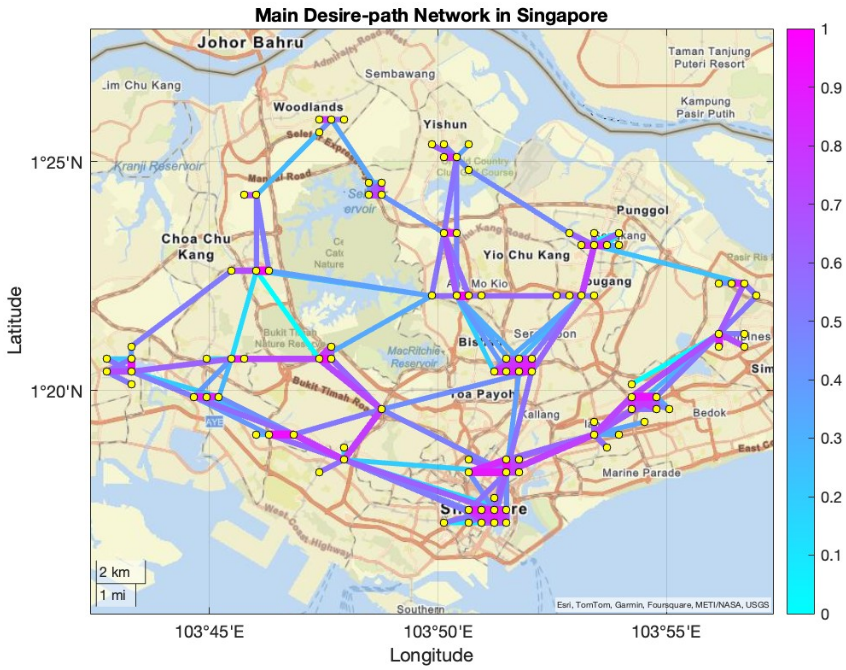


FIGURE 5: Extracted self-organized network in Singapore

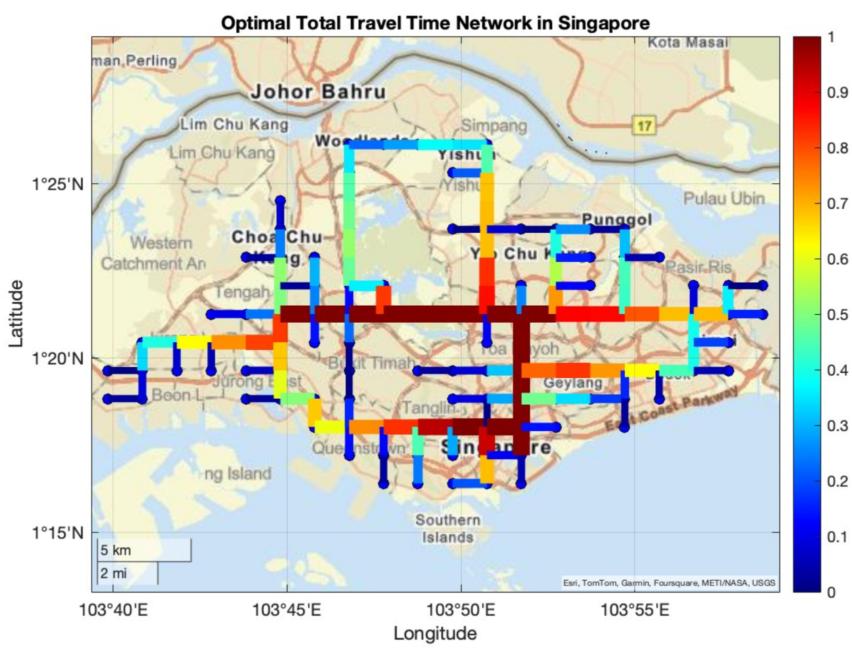
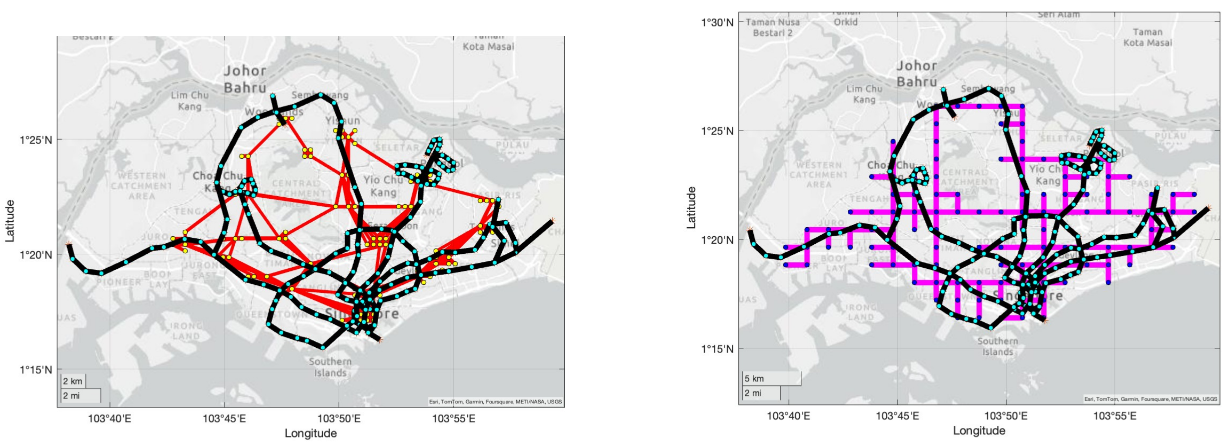


FIGURE 6: Globally optimal travel distance network in Singapore

Comparison between the obtained networks with real-world MRT transport network

As shown in Fig.7, our results show that the three networks have similar geometric silhouettes (as measured by the Hausdorff distance). The largest discrepancy is observed in the central part of the country, which contains a large nature reserve that prohibits the construction of a train line for environmental protection. Based on the OD data for the MRT network, we additionally apply concepts from optimal transport theory to explore the similarities and differences among the networks from a functional distribution perspective. Fig. 8 indicates the optimal transport plan between the self-organized network and the real-world MRT network, the globally optimal network and the real-world MRT network, of which the minimal transport cost is 17.07km and 26.35km respectively. This analysis reveals that the existing MRT network in Singapore is more similar to the self-organized network than to the globally optimal network. In other terms, the costs for modifying the existing system into the self-organized network are lower than those for modifying it into the globally optimal network. The primary differences in geometric shapes are observed near the MacRitchie Reservoir located at the central part. Due to environmental protection and geographical constraints, Singapore currently lacks a train line crossing the MacRitchie Reservoir. However, both the optimal network and the desired network suggest planning for this route. Therefore, to some extent, scheduling direct bus services could be considered to meet public demand while optimizing transportation efficiency.



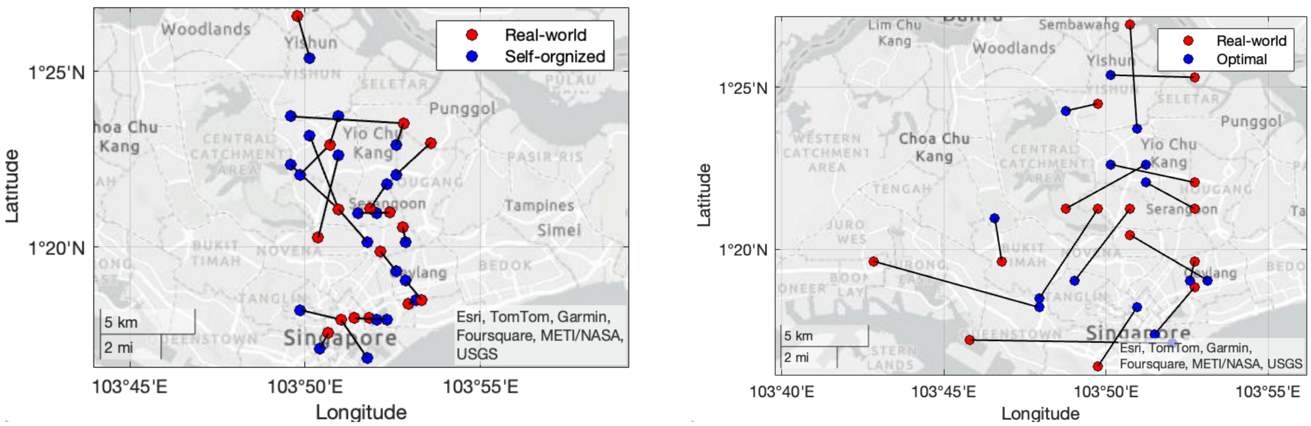
(a) Comparison of self-organized network (red) and real MRT network (black) in Singapore

(b) Comparison of globally optimal network (purple) and real MRT network (black) in Singapore

FIGURE 7: Comparison of self-organized, optimal and real-world networks

CONCLUSION

In this work, we introduce a data-driven modeling framework to study the similarities and discrepancies among three different transport networks including self-organized, globally optimal, and real-world transportation networks. The self-organized network balances human preferences for taking the shortest path and the propensity to share route segments with others. Our study shows that the Singapore MRT network is more similar to a self-organizing network than an optimal travel distance network. Our comparison also indicates the main differences between these three networks can be attributed to practical constraints, such as geographic limitations in building up



(a) Optimal transport plan from self-organized network to the real-world MRT network

(b) Optimal transport plan from globally optimal network to the real-world MRT network

FIGURE 8: Optimal transport plan between networks

links.

This study can be extended to several directions. First, practical constraints such as geographical limitations could be taken into consideration when deriving the self-organized network and optimal network. This definitely enables a better evaluation of the real-world network. Second, for the optimal network, it is also worth incorporating other objectives such as travel time and energy costs.

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AUTHOR CONTRIBUTIONS

The authors confirm their contribution to the paper as follows: study conception and design: T. Dong, M. Schläpfer; methodology: T. Dong, J. Zhou, M. Schläpfer; analysis and interpretation of results: T. Dong, J. Zhou, M. Schläpfer; original draft manuscript: T. Dong, J. Zhou, M. Schläpfer; draft manuscript review and editing: M. Schläpfer. All authors reviewed the results and approved the final version of the manuscript.

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